The transition of equation of state of effective dark energy in the DGP model with bulk contents

Shaoyu Yin, Bin Wang[*]

Department of Physics, Fudan University, 200433 Shanghai

Elcio Abdalla[†]

Instituto de Fisica, Universidade de Sao Paulo,
C.P.66.318, CEP 05315-970, Sao Paulo

Chi-Yong Lin[‡]

Department of Physics, National Dong Hwa University, Shoufeng, 974 Hualien

Abstract

We investigate the effect of the bulk contents in the DGP braneworld on the evolution of the universe. We find that although the pure DGP model cannot accommodate the transition of the effective equation of state of dark energy, once the bulk matter T_5^5 is considered, the modified model can realize the w_{eff} crossing -1. However this transition of the equation of state cannot be realized by just considering bulk-brane energy exchange or the GB effect while the bulk matter contribution is not included. T_5^5 plays the major role in the modified DGP model to have the w_{eff} crossing -1 behavior. We show that our model can describe the super-acceleration of our universe with the equation of state of the effective dark energy and the Hubble parameter in agreement with observations.

PACS numbers: 04.50.+h, 11.25.Wx, 95.36.+x, 98.80.-k

---

[*]Electronic address: wangb@fudan.edu.cn

[†]Electronic address: eabdalla@fma.if.usp.br

[‡]Electronic address: lcyong@mail.ndhu.edu.tw

I. INTRODUCTION

The accelerated expansion of our universe is one of the most important discovery in the last decade [1, 2, 3], having triggered plenty of efforts to understand and explain it. This phenomenon is in conflict with our common sense about attractive gravity. Within the framework of general relativity, the acceleration is attributed to the mysterious "dark energy" existing in our universe. The theoretical nature and origin of this dark energy are a source of much debate. Candidates suggested for this dark energy can be classified according to the behavior of their respective equation of state $w = P/\rho$. The cosmological constant, with $w = -1$, is located at a central position among dark energy models both in theoretical investigation and in data analysis [4]. In quintessence [5], Chaplygin gas [6] and holographic dark energy models [7], w always remains bigger than -1. The phantom models of dark energy have $w < -1$ [8]. Recent more accurate data analysis tells us a dramatic result, namely that the time varying dark energy gives a better fit than a cosmological constant and in particular, w can cross -1 around $z = 0.2$ from above to below [9]. Theoretical attempts towards the understanding of the w crossing -1 phenomenon have been suggested, including the model containing a negative kinetic scalar field and a normal scalar field [10], a single scalar field model [11], interacting holographic dark energy models [12] and others [13].

An alternative approach which does not need dark energy to explain the late-time acceleration is motivated by string theory via the brane-world scenarios. In this scenario our universe is a 3-d brane embedded in a space-time with extra dimensions. The cosmological evolution on the brane is described by an effective Friedmann equation incorporating non-trivially with the effects of the bulk onto the brane. The presence of the 5-d matter can interact with the matter contents on the brane and alter the cosmic expansion leading to a behavior resembling the dark energy. The cosmic evolution of the Randall-Sundrum(RS) braneworld [14] with energy exchange between brane and bulk has been studied [15, 16, 17, 18, 19]. In these models, due to the energy exchange between the bulk and the brane, the usual energy conservation law on the brane is broken and consequently it was found that the equation of state of the effective dark energy can experience the transition behavior [16, 17, 18, 19].

In string theory, in addition to the Einstein action, some higher derivative curvature

terms have been included to derive gravity. The combination of the Einstein-Hilbert and Gauss-Bonnet(GB) term constitutes, for 5D spacetimes, the most general Lagrangian to produce second-order field equations [20, 21]. The GB correction changes the bulk field equations and modifies the braneworld Friedmann equation. It influences the evolution of the universe in our brane. Effects of the GB correction on the RS braneworld have been studied in [19, 22].

In this paper we are going to concentrate on another braneworld model introduced by Dvali, Gabadadze and Porrati (DGP) [23], where the braneworld is embedded in the flat bulk with infinite extra dimensions. Considering that the graviton propagates into the extra dimension, and at large scale, gravity can become weaker due to its leakage, the DGP model can realize the accelerated expansion naturally. However for the pure DGP model, its effective equation of state never goes down to the phantom phase. Our main motivation here is to investigate the effects of the bulk contents in the DGP braneworld on the evolution of the universe and explore the possibility of the transition of equation of state if there are contributions from the bulk-related energy-momentum tensor components which has been observed in RS model[15, 16, 17, 18, 19]. The DGP model only with T_5^0 has been investigated in [24]. We are going to present a systematic and complete examination of the bulk effects including T_5^0 and T_5^5 terms on DGP model. Besides we will also study the modification on the brane evolution due to the GB correction together with bulk related energy-momentum tensor components. Influences of the GB correction on the pure DGP braneworld have been studied in [25, 26]. Although the effects of the GB correction term on the late time universe is small, we will see that it still plays an important role in the early time cosmic evolution. We will show that in the DGP model the bulk matter contribution T_5^5 plays a major role in accommodating the transition of equation of state, while the T_5^0 and GB correction alone cannot present a profile of the w_{eff} crossing -1 phenomenon found by observations.

The organization of the paper is the following: in section II we will give out the basic equation sets for the DGP model by considering different correction terms respectively. The bulk effects due to T_5^5 term will be shown in detail in section III. In section IV, we will consider the influence of the energy flow T_5^0 on the brane universe evolution. Conclusions and discussions will be presented in the last section.

II.   GENERAL EQUATIONS FOR DGP MODEL WITH GB CORRECTION

The DGP brane model with GB correction starts from the action

$$S = -\frac{1}{2\kappa^2}\int d^5X\sqrt{-g}(R - 2\Lambda_5 + \alpha L_{GB}) - \frac{1}{2\mu^2}\int d^4x\sqrt{-\widetilde{g}}(\widetilde{R} - 2\Lambda_4) + \int d^5\sqrt{-g}L_{EM}, \quad (1)$$

where κ and μ are related to the gravitational constants and the Planck masses for the bulk and brane as [27]:

$$\kappa^2 = 8\pi G_{(5)} = M_5^{-3}; \qquad \mu^2 = 8\pi G_{(4)} = M_4^{-2}, \quad (2)$$

respectively, Λ_5 and Λ_4 are cosmological constants for the bulk and brane. L_{EM} is the energy-momentum tensor and L_{GB} is the GB correction term in the form

$$L_{GB} = R^2 - 4R^{AB}R_{AB} + R^{ABCD}R_{ABCD}. \quad (3)$$

α is the coefficient of the GB term, which is positive, as required by string theory [20] and is generally considered to be very small. If we take $\alpha = 0$, Eq.(1) reproduces the pure DGP model [27]. Throughout the paper the capital letter are used to present the 5-d indices, while the Greek alphabet is used for 4-d brane case.

From the action one can obtain the field equation

$$G_{AB} + \Lambda_5 g_{AB} + 2\alpha H_{AB} = \kappa^2\{T_{AB} - [\frac{1}{\mu^2}(\widetilde{G}_{\mu\nu} + \Lambda_4\widetilde{g}_{\mu\nu}) + \widetilde{T}_{\mu\nu}]\delta(y_b)\delta_A^\mu\delta_B^\nu\}, \quad (4)$$

where $H_{AB} = RR_{AB} - 2R_A^C R_{BC} - 2R^{CD}R_{ACBD} + R_A^{CDE}R_{BCDE} - \frac{1}{4}g_{AB}L_{GB}$ is the second-order Lovelock tensor [21], $\delta(y_b)$ comes from the difference between the integration with 4-d metric and 5-d metric.

The energy-momentum tensor on the brane is assumed to be that of a perfect fluid,

$$\widetilde{T}_{\mu\nu} = (\rho + p)u_\mu u_\nu + p\widetilde{g}_{\mu\nu}, \quad (5)$$

where u_μ, ρ and p are the fluid velocity, energy density and pressure, respectively ($c = 1$ is used). The non-zero components related to the fifth dimension in the bulk energy-momentum tensor are supposed to be T_5^0 and T_5^5, whose role in the accelerated expansion will be studied in detail.

Generally, the metric in 5-d brane cosmology is written as

$$ds^2 = -n^2(t,y)dt^2 + a^2(t,y)\gamma^{ij}dx_idx_j + b^2(t,y)dy^2, \quad (6)$$

4

where y stands for the extra dimension orthogonal to the brane, and γ^{ij} is the maximally symmetric 3-d tensor. Then $\sqrt{-g} = b\sqrt{-\tilde{g}}$, thus $\delta(y_b) = \frac{\delta(y)}{b}$. From this metric the Einstein equation can be obtained directly. According to Eq.(4) one can obtain the Einstein tensor components as [28]

$$
\begin{aligned}
G_{tt} &= 3[n^2\Phi + \frac{\dot{a}}{a}\frac{\dot{b}}{b} - \frac{n^2}{b^2}(\frac{a''}{a} - \frac{a'}{a}\frac{b'}{b})], \\
G_{ty} &= 3(\frac{\dot{a}}{a}\frac{n'}{n} + \frac{a'}{a}\frac{\dot{b}}{b} - \frac{\dot{a}'}{a}), \\
G_{ij} &= \frac{a^2}{b^2}\gamma_{ij}[\frac{a'}{a}(\frac{a'}{a} + 2\frac{n'}{n}) - \frac{b'}{b}(\frac{n'}{n} + 2\frac{a'}{a}) + 2\frac{a''}{a} + \frac{n''}{n}] \\
&\quad -\frac{a^2}{n^2}\gamma_{ij}[\frac{\dot{a}}{a}(\frac{\dot{a}}{a} - 2\frac{\dot{n}}{n}) - \frac{\dot{b}}{b}(\frac{\dot{n}}{n} - 2\frac{\dot{a}}{a}) + 2\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b}] - k\gamma_{ij}, \\
G_{yy} &= 3[-b^2\Phi + \frac{a'}{a}\frac{n'}{n} - \frac{b^2}{n^2}(\frac{\ddot{a}}{a} - \frac{\dot{a}}{a}\frac{\dot{n}}{n})], \\
H_{tt} &= 6\Phi[\frac{\dot{a}}{a}\frac{\dot{n}}{n} + \frac{n^2}{b^2}(\frac{a'}{a}\frac{b'}{b} - \frac{a''}{a})], \\
H_{ty} &= 6\Phi(\frac{\dot{a}}{a}\frac{n'}{n} + \frac{a'}{a}\frac{\dot{b}}{b} - \frac{\dot{a}'}{a}), \\
H_{ij} &= 2a^2\gamma_{ij}\{\Phi[\frac{1}{n^2}(\frac{\dot{n}}{n}\frac{\dot{b}}{b} - \frac{\ddot{b}}{b}) - \frac{1}{b^2}(\frac{n'}{n}\frac{b'}{b} - \frac{n''}{n})] \\
&\quad + \frac{2}{a^2bn}[\frac{\dot{a}^2\dot{b}\dot{n}}{n^4} + \frac{a'^2b'n'}{b^4} + \frac{\dot{a}a'}{b^2n^2}(b'\dot{n} - \dot{b}n')] \\
&\quad - 2[\frac{1}{n^2}\frac{\ddot{a}}{a}(\frac{1}{n^2}\frac{\dot{a}}{a}\frac{\dot{b}}{b} + \frac{1}{b^2}\frac{a'}{a}\frac{b'}{b}) + \frac{1}{b^2}\frac{a''}{a}(\frac{1}{n^2}\frac{\dot{a}}{a}\frac{\dot{n}}{n} + \frac{1}{b^2}\frac{a'}{a}\frac{n'}{n})] \\
&\quad + \frac{2}{b^2n^2}[2\frac{\ddot{a}}{a}\frac{a''}{a} - \frac{\dot{a}^2}{a^2}\frac{n'^2}{n^2} - \frac{a'^2}{a^2}\frac{\dot{b}^2}{b^2} - \frac{\dot{a}'}{a}(\frac{\dot{a}'}{a} - 2\frac{\dot{a}}{a}\frac{n'}{n} - 2\frac{a'}{a}\frac{\dot{b}}{b})]\}, \\
H_{yy} &= 6\Phi[\frac{a'}{a}\frac{n'}{n} + \frac{b^2}{n^2}(\frac{\dot{a}}{a}\frac{\dot{n}}{n} - \frac{\ddot{a}}{a})],
\end{aligned}
\tag{7}
$$

where

$$
\Phi = \frac{1}{n^2}\frac{\dot{a}^2}{a^2} - \frac{1}{b^2}\frac{a'^2}{a^2} + \frac{k}{a^2}.
\tag{8}
$$

The dot denotes a derivative with respect to t, and the prime the derivative with respect to y. Without loosing generality, in the brane world scenario, one usually chooses the metric function $b(t,y) = 1$ and $n(t,0) = 1$ to simplify the calculation.

We choose the brane to be located at $y = 0$, and suppose the metric functions to be continuous at this point, but their first derivatives are discontinuous due to the energy-momentum tensor distribution on the brane. Furthermore, the geometry is supposed to display a Z_2-symmetry around $y = 0$, thus $a'(0_+) = -a'(0_-)$ and $n'(0_+) = -n'(0_-)$. If one integrates the field equation for the tt and ij components at the infinitesimal region near

5

$y = 0$, only those terms in the metric with a'' or n'' and the energy-momentum distribution on the brane can remain. Then the differences of a' and n' on both sides of the brane, say, $a'(0_+) - a'(0_-) \equiv 2a'(0_+)$ and $n'(0_+) - n'(0_-) \equiv 2n'(0_+)$ can be obtained. For simplicity, throughout the paper we use a' and n' to stand for $a'(0_+)$ and $n'(0_+)$, and in the equation on the brane all bulk terms are taken with their values at $y = 0_+$. From G_{tt}, G_{ij}, H_{tt}, H_{ij} in Eq.(7), we find that a' and n' satisfy the following equations:

$$\frac{a'}{a}\{-3 + 4\alpha[\frac{a'^2}{a^2} - 3(\frac{k}{a^2} + \frac{\dot{a}^2}{a^2})]\} = \frac{\kappa^2}{2\mu^2}[\mu^2\rho - 3(\frac{k}{a^2} + \frac{\dot{a}^2}{a^2})]; \tag{9}$$

$$n' + \frac{2a'}{a} + 4\alpha[n'(\frac{k}{a^2} - \frac{a'^2}{a^2} + \frac{\dot{a}^2}{a^2}) + \frac{2a'}{a}(\frac{\ddot{a}}{a} - \frac{\dot{a}\dot{n}}{a})] = \frac{\kappa^2}{2\mu^2}(\mu^2 p + \frac{k}{a^2} + \frac{\dot{a}^2}{a^2} - \frac{2\dot{a}\dot{n}}{a} + \frac{2\ddot{a}}{a}). \tag{10}$$

Generally, one can solve these equations and substitute the results of a' and n' into the field equation for the ty and yy components to obtain the continuity equation and the effective Friedmann equation. But before that, it is helpful to notice that the function Φ we introduced in Eq.(8) satisfies

$$\widetilde{\Phi}' = \frac{2\kappa^2}{3}(\Lambda_5 - T_0^0)a^3 a' - \frac{2\kappa^2}{3}T_5^0 a^3 \dot{a}, \tag{11}$$

$$\dot{\widetilde{\Phi}} = \frac{2\kappa^2}{3}(\Lambda_5 - T_5^5)a^3 \dot{a} - \frac{2\kappa^2}{3}\frac{n^2}{b^2}T_5^0 a^3 a', \tag{12}$$

where $\widetilde{\Phi} \equiv (\Phi + 2\alpha\Phi^2)a^4$. From (12), if $T_0^0 = 0$ and T_5^5 has a proper *ansatz*, such as $T_5^5 = \frac{F}{\kappa^2}a^\nu$, $\widetilde{\Phi}$ can be obtained analytically by an integration with respect to t:

$$\widetilde{\Phi} = \frac{\kappa^2}{6}\Lambda_5 a^4 - \frac{2Fa^{\nu+4}}{3(4+\nu)} + C, \tag{13}$$

where C is an integration constant. For the case without GB term, the solution of Φ is simply

$$\Phi = \frac{\kappa^2}{6}\Lambda_5 - \frac{2Fa^\nu}{3(4+\nu)} + \frac{C}{a^4}, \tag{14}$$

where the term $\frac{C}{a^4}$ is usually referred to the dark radiation [29]. For the case with GB correction, solutions of Φ are,

$$\Phi = \frac{-(4+\nu) \pm \sqrt{(4+\nu)^2}\sqrt{1 + 8\alpha(\frac{\kappa^2\Lambda_5}{6} + \frac{C}{a^4} - \frac{2Fa^\nu}{3(4+\nu)})}}{4\alpha(4+\nu)}, \tag{15}$$

while only one solution with finite $\alpha \to 0$ limit can be taken. For $\nu > -4$ the solution reads

$$\Phi = \frac{-1 + \sqrt{1 + 8\alpha(\frac{\kappa^2\Lambda_5}{6} + \frac{C}{a^4} - \frac{2Fa^\nu}{3(4+\nu)})}}{4\alpha}. \tag{16}$$

When $\alpha \to 0$, Eq.(16) goes back to Eq.(14).

From the definition of Φ in Eq.(8), we see that a' can be expressed in terms of $\dot{a}$ once Φ is obtained. Integrating the equation of tt component around $y = 0$ and substituting a' in terms of Φ and $\dot{a}$, we can finally arrive at the equation for the Hubble parameter $H(t) \equiv \frac{\dot{a}}{a}$,

$$(H^2 + \frac{k}{a^2} - \Phi)[1 + \frac{8\alpha}{3}(H^2 + \frac{k}{a^2} + \frac{\Phi}{2})]^2 = \frac{r^2}{4}[H^2 + \frac{k}{a^2} - \frac{\mu^2}{3}(\rho + \Lambda_4)]^2, \qquad (17)$$

where $r = \kappa^2/\mu^2$ is the DGP crossover radius.

For the sake of simplicity and clarity in the following discussion, we give the simplifications we are going to use. We will neglect the cosmological constant, $\Lambda_5 = \Lambda_4 = 0$, since the effect of the cosmological constant can be included in ρ and p. We will apply our discussion to the flat universe with $k = 0$. Besides, we will employ dimensionless notations in the following calculation by defining

$$x = \frac{H^2}{H_0^2},$$

$$z = \frac{a_0}{a} - 1,$$

$$u = \frac{\mu^2 \rho}{3H_0^2},$$

$$y = \frac{\Phi}{2H_0^2},$$

$$n = \frac{1}{H_0^2 r^2},$$

$$m = \frac{8H_0^2}{3}\alpha,$$

$$X = \frac{a_0^2}{H_0^2(4 + \nu)}F,$$

$$\widetilde{X} = \frac{a_0^2}{H_0^2}F,$$

$$M = \frac{3}{2H_0^2 a_0^4}C, \qquad (18)$$

where a_0 and H_0 are the present values of the scale factor and the Hubble parameter, z is the redshift and $u/x = \frac{\mu^2 \rho}{3H^2}$ is the proportion of matter in the total effective energy density.

Using dimensionless notations, the expression for the solution of Φ becomes

$$\Phi = 2H_0 \frac{-X(1 + z)^{-\nu} + M(1 + z)^4}{3}, \qquad (19)$$

for the case without the GB correction; if the GB term is included, it reads

$$\Phi = 2H_0 \frac{-1 + \sqrt{1 + 2m(-X(1 + z)^{-\nu} + M(1 + z)^4)}}{3m}. \qquad (20)$$

7

The equation for H^2, Eq.(17), turns into an equation for x [26]

$$4n(x - 2y)[1 + m(x + y)]^2 = (x - u)^2. \tag{21}$$

If T_5^0 is nonzero, Φ cannot be solved analytically and we can not take advantage of the simplicity discussed above. To obtain the equation for H^2, we need to substitute the solutions of a' and n' into the equation of yy component, and obtain $H(t)$ through onerous calculations. The acceleration of the scale factor can be written as $\ddot{a} \equiv a(H^2 + \dot{H})$. By choosing the proper *ansatz* of T_5^0 and expressing the result of $\rho(t)$ as a function of $a(t)$, we can generally obtain the equation as a nonlinear ordinary differential equation of $H(t)$, combining with the unknown function $a(t)$. To solve such a problem in RS model [16, 18] the authors introduced new effective fields related to $H(t)^2$ and separated the equation into two equations, both of which are solvable separately. But for the DGP model, due to the extra 4-d intrinsic curvature terms, the highest order of $H(t)$ is 4, rather than 2 in the RS model. With the GB correction, the order goes up to 8. In Ref.[24] when just nonzero T_5^0 was included in the pure DGP model, the author solved the problem by introducing the concept of "fix point" and setting $\rho(t)$ and the auxiliary field time-independent. Generally, we do not hope to obtain any analytical solution for such a nonlinear ordinary differential equation. We will count more on the numerical calculations. Considering nonzero T_5^0 and T_5^5 components, our problem is general and complicated. We will present a general way to solve the problem.

We have two time-dependent functions, $H(t)$ and $a(t)$, in the same equation. Considering that in the big-bang cosmology the flat universe is expanding monotonically, $a(t)$ is a monotonic function of t, and $H(t)$ can be written as $H(a)$. For the convenience we will use the dimensionless redshift z and write $H(t)$ as

$$\dot{H}(t) = -\frac{H_0^2(1 + z)}{2}\frac{dx(z)}{dz}. \tag{22}$$

Substituting all the dimensionless notations into the equation of the yy component and expressing the results until the linear order in α, the equation for $H(t)$, or equally, $x(z)$, is

$$
\begin{aligned}
0 =\ & -mx^4 + m[24n + u + (1 + z)x']x^3 \\
& + [-16n + 48mn^2 - 12mnu + 3mu^2 - 3m(6n + u)(1 + z)x']x^2 \\
& + \{64n^2 + 8nu - 12mnu^2 - 5mu^3 + [8n(1 - 3mn) + 3mu(8n + u)](1 + z)x'\}x
\end{aligned}
$$

8

$$+ \ [8nu^2 + 2mu^4 - \{16n^2 + u[8n + mu(6n + u)]\}(1+z)x']$$
$$+ \ \frac{32n^2(1+z)^{-\nu}}{3}\widetilde{X}, \tag{23}$$

where the prime here is the derivative with respect to z, x and u are functions of z, and we have taken $T_5^5 = \frac{F}{\kappa^2}a^\nu$. It is to be noted that since we don't need to analytically integrate T_5^5 with the term $a^3\dot{a}$ as did in Eq.(12), we can in principle use any form of T_5^5 as a function of a. Eq.(23) is a nonlinear differential equation of $x(z)$.

From the equation of the ty component, assuming $b(t,y) = 1$, we get

$$3(1 + 4\alpha\Phi)(\frac{\dot{a}}{a}\frac{n'}{n} - \frac{\dot{a}'}{a}) = T_{05}. \tag{24}$$

Taking the value of each term in this equation at $y = 0_+$ and substituting the solution of a' and n', we can find that the left hand side of this equation is simply $\frac{1}{2}(\dot{\rho} + 3H(\rho + p))$. If $T_5^0 = 0$, it is the conservation of energy on the brane. If $T_5^0 \neq 0$, it acts as the energy flow between the brane and the bulk,

$$\dot{\rho} + 3H(\rho + p) = -2T_5^0. \tag{25}$$

Here T_5^0 has a sign difference as compared to T_{05} due to the metric term $g_{tt}|_{y=0} = -n(t,0) = -1$.

Setting the *ansatz*, $T_5^0 = fHa^s$, the equation for ρ can be solved analytically. Expressing $\rho(t)$ as $\rho(a)$, we have

$$\frac{d\rho}{da} + \frac{3(1+w)\rho}{a} + 2fa^{s-1} = 0, \tag{26}$$

with the solution

$$\rho = a^{-3-3w}C_1 - \frac{2fa^s}{3 + 3w + s}, \tag{27}$$

where C_1 is an integration constant. For the cold matter on the brane $w = 0$, the first term on the right-hand-side is proportional to a^{-3} and the second term could be attributed to the effective dark energy. Eq.(27) can be expressed by using the dimensionless notation $u(z)$,

$$u(z) = P(1+z)^{-s} + \Omega_{m0}(1+z)^3, \tag{28}$$

where $P = -\frac{2\mu^2 a_0^s}{3(3+s)H_0^2}f$, and $\Omega_{m0} = \frac{\mu^2 C_1}{3H_0^2 a_0^3} = \frac{8\pi G}{3H_0^2}\rho_0$ is the present ratio of conservative matter in the total energy density of the universe, ρ_0 is the density of the conservative matter today. There is a strong constraint on the value of dimensionless parameter P. Since the matter portion of the total energy density should be in the range $[0,1]$, thus $0 \leq \frac{u(z)}{x(z)} \leq 1$. Here we

will use the fitting results on the WMAP data [30] and take $\Omega_{m0} = 0.28$ in our calculation. Of course, that fitting is from a different model, but the generally accepted values of Ω_{m0} are all close to this value, and the small variation of this value will not change the qualitative conclusion of our calculation. At the present moment $z = 0$, we have $-0.28 \leq P \leq 0.72$. Any solution with P out of this range is physically unreasonable.

To describe the effect of the effective dark energy, we can define the effective equation of state [31]:

$$w(z)_{eff} \equiv -1 + \frac{1}{3}\frac{d\ln\delta H^2}{d\ln(1+z)}, \tag{29}$$

where $\delta H^2 \equiv H(z)^2 - \Omega_{m0}(1+z)^3 H_0^2$. w_{eff} can be expressed by using the dimensionless parameters as

$$w(z)_{eff} = -1 + \frac{(1+z)\frac{dx(z)}{dz} - 3\Omega_{m0}(1+z)^3}{3x(z) - 3\Omega_{m0}(1+z)^3}. \tag{30}$$

The subscript eff indicates that the effect similar to the dark energy on the brane comes from the bulk contribution. Another important quantity is the deceleration parameter q

$$q \equiv -\frac{\ddot{a}a}{\dot{a}^2} = \frac{1}{2} + \frac{3}{2}w(z)_{eff}(1 - \frac{\Omega_{m0}(1+z)^3}{x(z)}), \tag{31}$$

which will also be used in the following discussion of the expansion of our universe.

III.   CALCULATION AND DISCUSSION WITHOUT CONSIDERING THE T_5^0 TERM

For the case without T_5^0 term, we can solve the equation Eq.(21) by substituting Φ from Eq.(20) and u from Eq.(28) but with $P = 0$. We can put the result into Eq.(30) and Eq.(31) to examine the behavior of the effective dark energy.

In Eq.(21), if there is no GB correction, the equation for x is quadratic, thus we have two solutions. For the case $\Phi = 0$, they recover the two branches of pure DGP model, DGP(+) and DGP(-): $x = 2n + (1 + z)^3\Omega_{m0} \pm 2\sqrt{n^2 + n(1+z)^3\Omega_{m0}}$. Since only the DGP(+) solution has late-time self-accelerating behavior [25], we will concentrate our discussion on this solution. When GB correction is added, Eq.(21) becomes a cubic equation and has three roots for x, two of which correspond to DGP(+) and DGP(-) in $\alpha \to 0$ limit, and the third one diverges when $\alpha \to 0$. For comparison, we also study the solution with DGP(+) limitation for that case in this work.

To compare our model description on the evolution of the universe with the observation, we have several constraints to meet. At present we have $x(z = 0) \equiv H(t = 0)^2/H_0^2 \equiv 1$. For the effective equation of state, we require $w_{eff}(z = 0.2) = -1$ and $w_{eff}(z = 0) = -1.06$ [9]. We will use these constraints to refine our model parameters. Assuming $T_5^0 = 0$, we have parameters such as n (corresponding to the DGP crossover radius), m (corresponding to the GB correction), M (corresponding to the dark radiation), X and ν (relating to T_5^5 form). We will focus on whether we can accommodate the w_{eff} crossing -1 by including bulk related energy-momentum tensor and the GB correction, which cannot be realized in the pure DGP model.

Since we have the parameter ν in the exponential, the equations are not polynomial. We will use *FindRoot* in our calculation which will raise the problem about the choice of the initial values. To avoid the possibility of failing to find the solution, we will try different initial values in the *FindRoot*. We find that the solution is not strongly dependent on the initial values so that we are confident that our results are almost the whole collection of all possible solutions to the equations we deal with.

To see the consistency of our results with observation, we will plot the Hubble parameter and compare with the observational $H(z)$ data as shown in Table 1. It is interesting to note that all the cases which can accommodate the equation of state transition can fit well the observational $H(z)$ data. Remembering that the w_{eff} crossing -1 was observed in the SNIa data fitting containing an integration effect in the luminosity distance, while the Hubble parameter does not suffer from this integrated over effect, the Hubble parameter data can present a complementary and consistent check for our model.

Now we list out our results step by step. In the following results, all the numbers obtained in numerical calculation are expressed only with 3 digits after decimal unless for the cases where more digits are necessary to be shown.

1. DGP+M

z	0.09	0.17	0.27	0.40	0.88	1.30	1.43	1.53	1.75
H(z) (km/s/Mpc)	69	83	70	87	117	168	177	140	202
1σ uncertainty	± 12	± 8.3	± 14	± 17.4	± 23.4	± 13.4	± 14.2	± 14	± 40.4

TABLE I: The observational $H(z)$ data [32, 33].

In this step we consider the DGP model with the dark radiation, where M denotes the dark radiation. Now we have two free parameters n and M and we will use two constraints: $x(0) = 1$ and $w_{eff}(0.2) = -1$ to see whether the dark radiation can help to realize the w_{eff} crossing -1. Actually n and M can be solved by using these constraints as $n = 0.157$ and $M = 0.263$. But using these values of n and M, the w_{eff} behavior is not good. w_{eff} crosses -1 at $z = 0.2$ from below to up as shown in Fig.1, and the present value is $w_{eff} = -0.950$. This is not in consistent with the observation, especially the transition behavior.
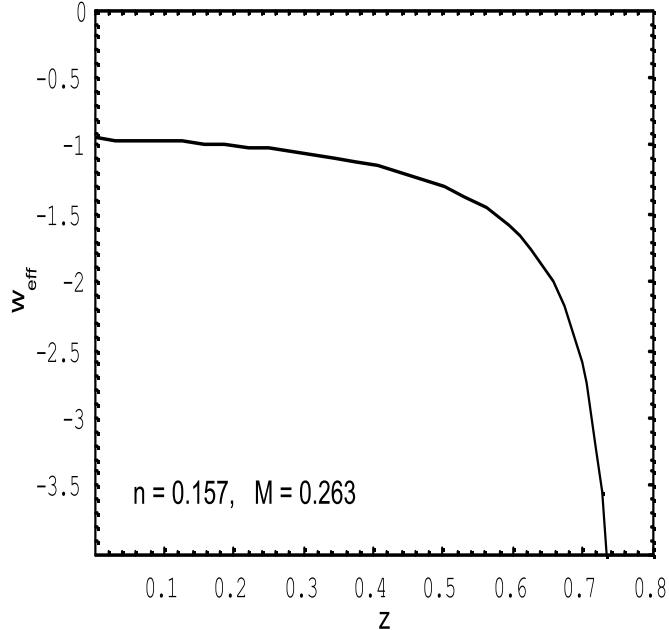
FIG. 1: The w_{eff} curve as function of z in the case DGP+M. The behavior of w_{eff} is bad since it crossed -1 at $z = 0.2$ from below to above.

2. DGP+GB+M

Including the GB correction, we have three free parameters now, such as n, m and M. If we apply all three constraints ($x(0) = 1$, $w_{eff}(0.2) = -1$ and $w_{eff}(0) = -1.06$), the solution is complex ($n = -0.030 + 0.093i$, $m = -0.002 - 0.007i$ and $M = 6.340 + 6.244i$). If we apply only two constraints ($x(0) = 1$ and $w_{eff}(0.2) = -1$) and search n in a big range $0.001 \leq n \leq 5$, the similar result to that in case 1 appears: w_{eff} crosses -1 from below to above and m is negative. One solution is shown in Fig.2a. We note that there is a singularity about $z = 1.246$ in the curve of w_{eff}. This singularity comes from the definition

in Eq.(29), we see that when $\delta H^2 \equiv H(z)^2 - \Omega_{m0}(1+z)^3 H_0^2 \leq 0$, the logarithm is not well defined, but one can still calculate the w_{eff} through the simplified expression in Eq.(30). In Fig.2b we show the relation between H^2/H_0^2 and the matter component $\Omega_{m0}(1+z)^3$. We see that beyond the redshift $z = 1.246$ the matter component is overweight, so $\delta H^2 < 0$, which means the effective dark energy component is negative. This is obviously unreasonable, and at least it shows that the model fails in explaining the universe before that redshift. In this work we concentrate on those solutions with $w_{eff}(z)$ free of singularity.
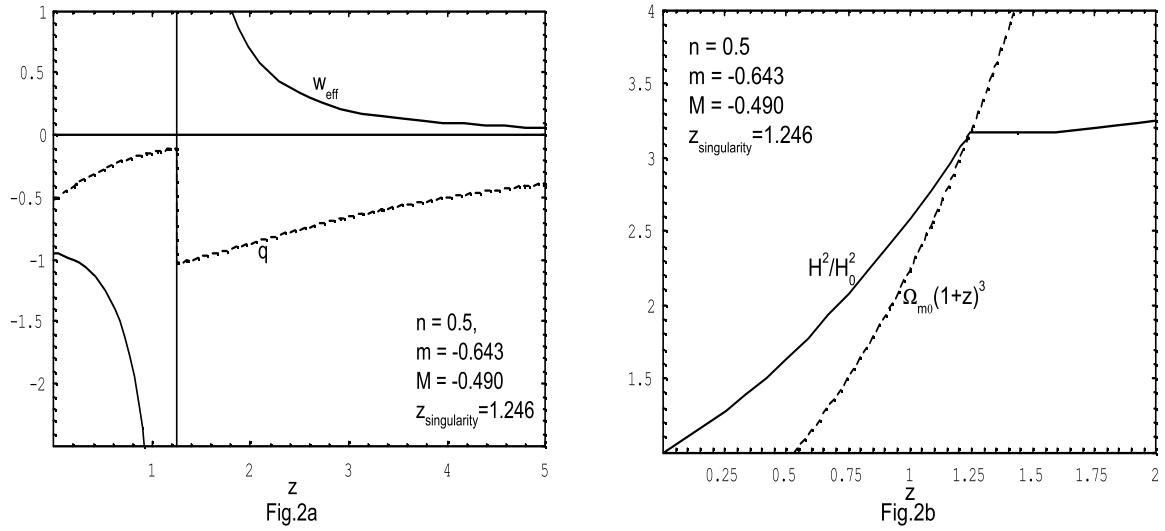
Fig.2a

Fig.2b

FIG. 2: In the case DGP+GB+M, w_{eff} and q as functions of z are shown in Fig.2a. The behavior is not favored as in Fig.1. Singularity is observed at $z = 1.246$. In Fig.2b, relation between H^2/H_0^2 and the matter component $\Omega_{m0}(1+z)^3$ is shown. When $z > 1.246$, $\delta H^2 \equiv H(z)^2 - \Omega_{m0}(1+z)^3 H_0^2$ becomes negative, which breaks the definition of w_{eff} in Eq.(29).

3. DGP+T_5^5

Now we include the bulk related energy-momentum tensor T_5^5. In this case we have three free parameters (n, X and ν) and we are going to employ all three constraints ($x(0) = 1$, $w_{eff}(0.2) = -1$ and $w_{eff}(0) = -1.06$). We can find the solution $n = 0.046$, $X = 2.729$ and $\nu = 0.948$. The curves of w_{eff}, q and H v.s. redshift z are shown in Fig.3a and Fig.3b respectively. In plotting Fig.3b, we have used $H_0 = 72 km/s/Mpc$ [34]. It is interesting to find that parameters adjusted to meet the requirement of w_{eff} crossing -1 and its value at the present moment automatically fit well to the $H(z)$ data. Recalling that the transition

13

behavior of w_{eff} results from the SN data analysis containing integration in the luminosity distance, while the Hubble parameter is not integrated over, which persists fine structure highly degenerated in the luminosity distance, the simultaneous satisfaction of the w_{eff} behavior and the $H(z)$ data gives complementary and consistent check of the viability of our model.
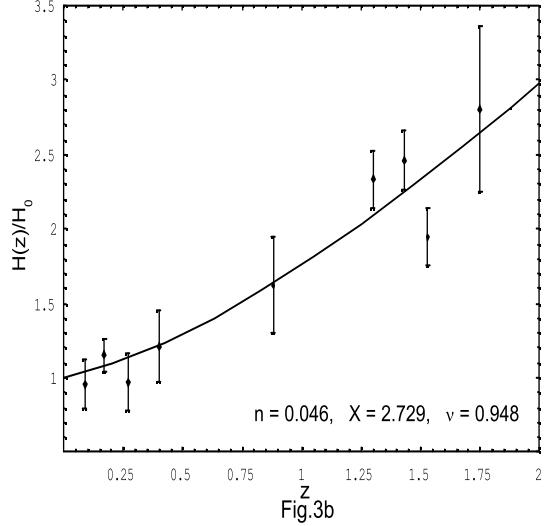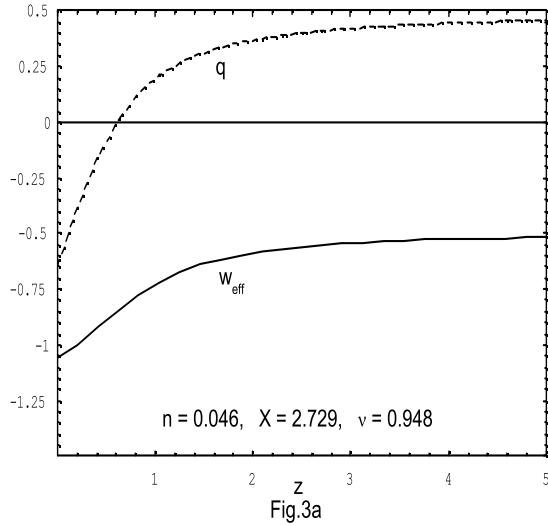

FIG. 3: w_{eff} and q v.s. z (Fig.3a) and $H(z)$ curve (Fig.3b) in DGP+T_5^5 case. We see that $H(z)$ curve fits the data quite well.

4. DGP+GB+T_5^5

Here we include the GB correction based on the case discussed above. We now have four free parameters n, m, X and ν. Employing constraints $(x(0) = 1, w_{eff}(0.2) = -1$ and $w_{eff}(0) = -1.06)$ and searching through $0.001 \leq n \leq 5$, we see that m, X and ν can either be negative or positive, but the latter two are always with the same sign. m decreases with the increase of n and can be positive only when $n < 0.05$. The w_{eff} curve has no singularity for positive m, but contains singularity when $m < 0$ (few solutions without singularity have been found with negative m, but then ν becomes smaller than -4, which conflicts with our simplification assumption discussed above). We are interested in the positive m, since GB correction coefficient α is always positive, the singularity-free curves of $w_{eff}(z)$ and $q(z)$ with positive solution $m = 0.025$ are shown in Fig.4a. When the GB correction is considered, w_{eff} appears more different at larger z if compared to the result without the GB correction.

Since the GB effect is only important in the early universe, its stronger modification to the w_{eff} at bigger redshift is natural. In Fig.4b, we plotted the $H(z)$ curve by using the same adjusted parameter from the constraints on the equation of state, and we see again that in the model when the w_{eff} requirement is met, the H parameter automatically fits the data, which gives the consistent check of the model.
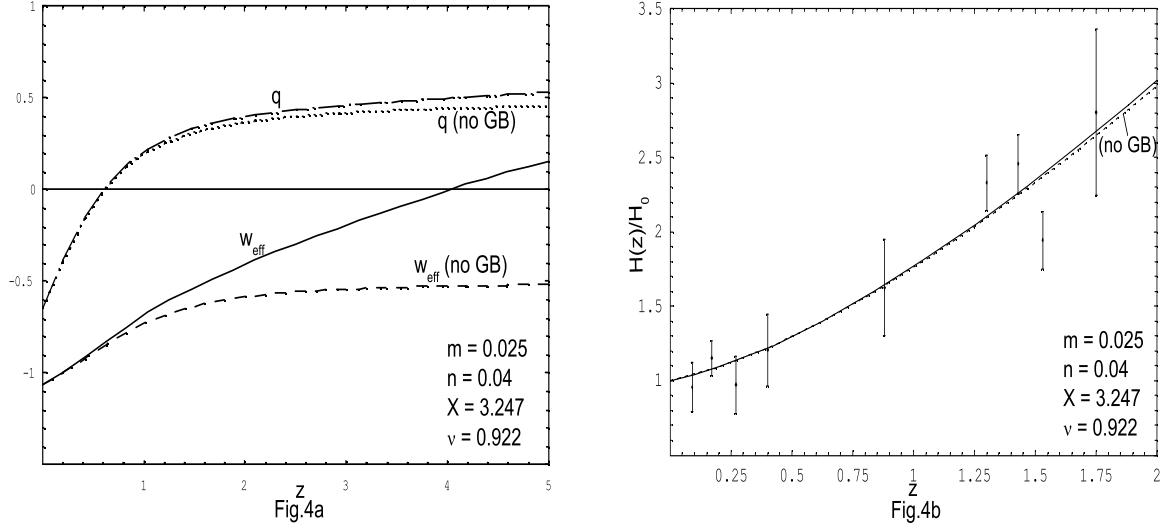

FIG. 4: $w_{eff}(z)$ and $q(z)$ curves (Fig.4a) and $H(z)$ curve (Fig.4b) in DGP+GB+T_5^5 case. Comparisons with the results without GB corrections have been shown. Differences from the case without the GB correction become bigger at higher redshift.

5.DGP+T_5^5+M

Based on case 3, we include the dark radiation contribution. We now have four parameters, namely n, X, ν and M. Employing three constraints ($x(0) = 1$, $w_{eff}(0.2) = -1$ and $w_{eff}(0) = -1.06$), and searching n in the range $0.001 \leq n \leq 5$, we find that the solutions exist only when $n < 0.36$. We see that M can either be positive or negative: for the negative M, w_{eff} never decreases with the increase of z within $z < 5$; while for the positive M, w_{eff} drops at large z, and singularities of $w_{eff}(z)$ and $q(z)$ appear within $z < 5$. Pictures of these two cases are shown in Fig.5a and Fig.5b. It is also observed that with the increase of n, the positive value of M becomes bigger and the singularity appears at smaller z.

6. DGP+GB+T_5^5+M

Now we have all five parameters: n, m, X, ν and M and three constraints to be used

15
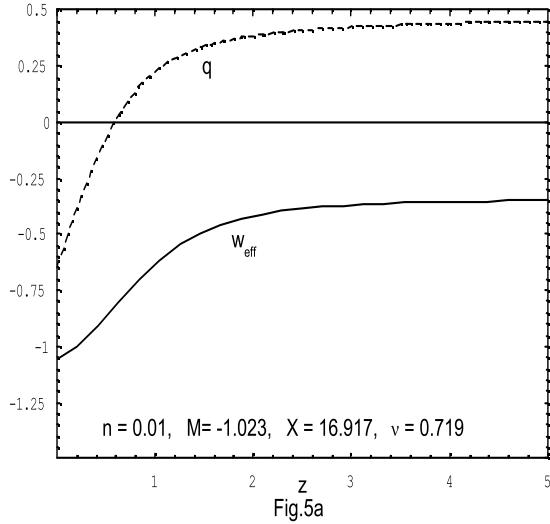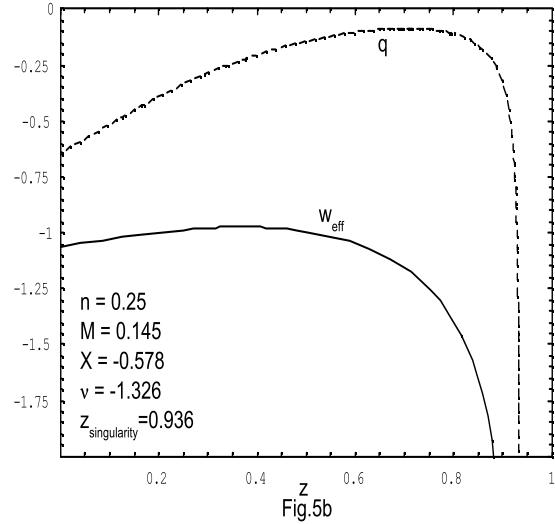
FIG. 5: We show in Fig.5a the $w_{eff}(z)$ and $q(z)$ curves in DGP+T_5^5+M case, where M is negative and the curves are singularity free; in Fig.5b, M is positive and the curves have a singularity at $z = 0.936$.

$(x(0) = 1$, $w_{eff}(0.2) = -1$ and $w_{eff}(0) = -1.06)$. First, we try to set m small, e.g. 10^{-9}, and choose one solution obtained in case 5, but the *FindRoot* command cannot help to get a solution to meet all three constraints, even though the initial parameters are set closely to those in case 5. This means that the solution is sensitive to α, though α (or m) is small. Next we search the nonsingular solutions within the parameters' ranges $0.001 \leq m \leq 0.02$, $0.01 \leq n \leq 0.1$, the curves are not quite different from what we have seen in case 3 and case 5. We show one of the solutions in Fig.6a and Fig.6b.

IV. CALCULATION AND ANALYSIS CONSIDERING THE T_5^0 TERM

The study with nonzero T_5^0 becomes more complicated. In order to solve the effective Friedmann equation, or equivalently, to obtain $x(z)$, we have to use the differential equation (23) with the full expression of u in Eq.(28). The free parameters we have now are n, m, $\widetilde{X}$, ν, P and s. Here we do not have the explicit dark radiation term M, since we do not make the integration to get Φ. In the calculation, we have to solve the differential equation, where the boundary condition $x(z = 0) = 1$ is needed. The effect of the dark radiation is

16
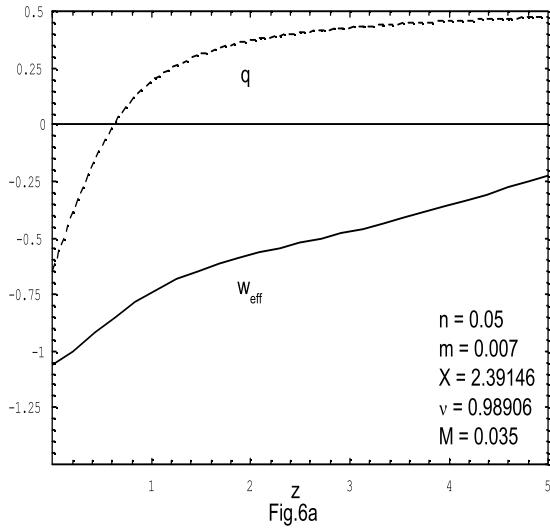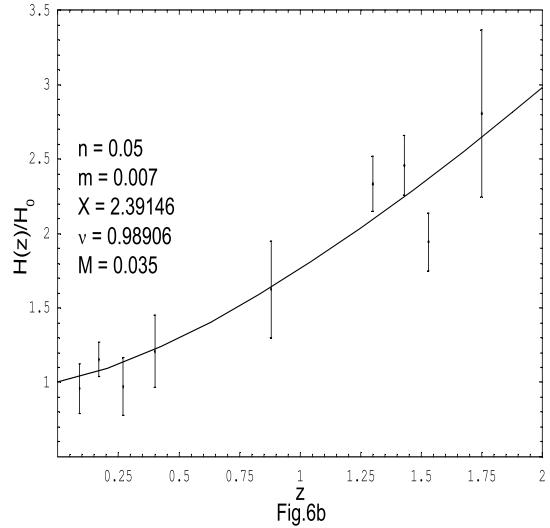
FIG. 6: $w_{eff}(z)$ and $q(z)$ curves (Fig.6a) and $H(z)$ curve (Fig.6b) in DGP+GB+T_5^5+M case.

reflected in the boundary condition of x. Solving equations when $T_5^0 = 0$, the result is the same as the case when dark radiation term M appeared in the last section. We will still use two constraints on w_{eff}, which are $w_{eff}(0.2) = -1$ and $w_{eff}(0) = -1.06$.

Eq.(23) is a differential equation of $x(z)$, where free parameters are involved. More efforts are needed to solve the equation numerically. Here we try to employ the self-consistent method. First we get $x(z)$ solved with the chosen initial values of parameters, then we substitute the numerical result into the expression of $w_{eff}(z)$. The term $\frac{dx(z)}{dz}$ in the expression of w_{eff} can be replaced by the function of $x(z)$ using Eq.(23), thus we can write w_{eff} into a function of $x(z)$. Substituting the solution of $x(z)$, w_{eff} becomes a function of z and we can solve the parameters with the constraints $w_{eff}(0.2) = -1$ and $w_{eff}(0) = -1.06$. If the result is not consistent, we substitute the results back to $x(z)$ as new initial values until convergence is finally arrived. A proper choice of the initial parameters is crucial to obtain a convergent result, we usually do the iteration with many different choices within quite a large reasonable parameter space.

Now we show the results we have obtained. First, to show the consistent and efficient of our numerical calculation, we turn off the contribution of T_5^0 to recover corresponding cases in section III.

7. DGP+T_5^5 (employing Eq.(23) to solve the problem numerically)

17

We have three parameters n, $\widetilde{X}$ and ν in this case. Searching within ranges $-100 \le \widetilde{X} \le 100$ and $-4 \le \nu \le 100$ by setting $n = 0.01$, we can find the solution ($\widetilde{X} = 79.824$, $\nu = 0.719$), which is singularity free at least for $z < 5$. This result can be compared with that in case 5 (DGP+T_5^5+M). Here the dimensionless parameter for T_5^5 term is $\widetilde{X}$, which differs from X employed in case 5 with a factor $4 + \nu$ as shown in Eq.(18). Taking this into account, the solution for X is 16.917, which is just the result in case 5 with $n = 0.01$, where the solution in case 5 reads $X = 16.917$, $\nu = 0.719$ and with the additional parameter $M = -1.023$, see Fig.5a. They coincide, although they are obtained by completely different methods. This shows the correctness of the self-consistent method we used and also demonstrates the equivalence of the boundary condition in differential equation and the extra freedom of the integration constant.

8. DGP+GB+T_5^5 (employing Eq.(23) to solve the problem numerically)

We have now four parameters n, m, $\widetilde{X}$ and ν. Within ranges $-100 \le \widetilde{X} \le 100$, $0 \le \nu \le 5$ by setting $n = 0.05$ and $m = 0.007$ (which are the values used in case 6 in Fig.6a and Fig.6b), we can find the solution $\widetilde{X} = 11.93120$ and $\nu = 0.98898$. Considering the difference between the dimensionless notations, this corresponds to $X = 2.39151$ and $\nu = 0.98898$, which is a bit different from those directly obtained in case 6 as $X = 2.39146$, $\nu = 0.98906$. This small difference is due to the approximation we have taken in Eq.(23), where the expansion on α is kept only to the linear order. So when $\alpha \ne 0$, the equation systems in case 6 and case 8 are not exactly the same. We can see that the difference between the curves in Fig.7a, 7b and those in Fig.6a, 6b lies in large z region, which shows that the effect of GB correction is important in the large redshift era.

To demonstrate more explicitly the effect of GB term, we calculate in this case with different fixed values of m. We shut down the freedom of ν by setting $\nu = 1$ in order to show the effect of GB term more clearly. The results are shown in Fig.8, where we see clearly that the GB term only changes the property in the early universe.

9. DGP+T_5^0

We turn on the T_5^0 effect to consider the energy exchange between the bulk and the brane. In this case free parameters are n, P and s. Searching in the range $0.01 \le n \le 0.2$ with $-0.28 \le P \le 0.72$ and $-100 \le s \le 100$ as initial tries, we find that for $n \le 0.15$, solutions can be found, but all require $P < -0.28$, which is forbidden as we discussed. This tells us that T_5^0 alone with the simple form $T_5^0 = fHa^s$ cannot lead to the expected behavior of the
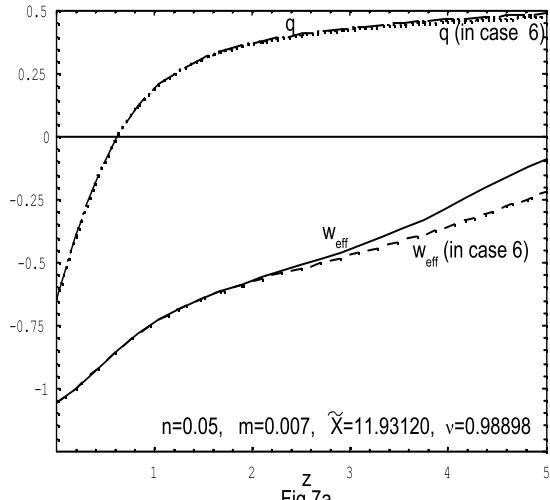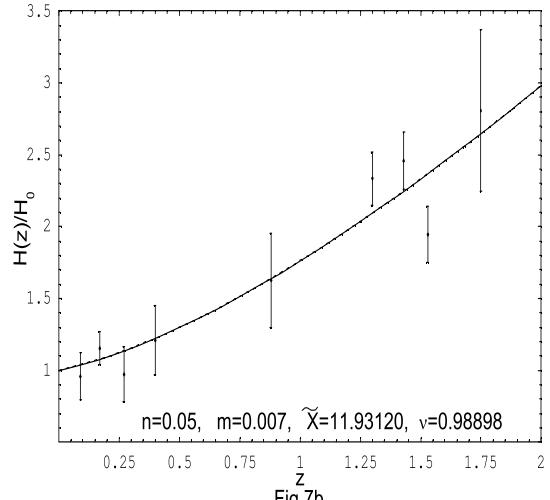
FIG. 7: $w_{eff}(z)$ and $q(z)$ curves (Fig.7a) and $H(z)$ curve (Fig.7b) in DGP+GB+T_5^5 case. This result is calculated from the equation considering the T_5^0 term, which is not quite different from Fig.6 of case 6, except the curves lie a little higher than the curves in Fig.6a at large z.

effective equation of state.

10. DGP+GB+T_5^0

We have four parameters n, m, P and s, one more than those in the previous case. But after searching in ranges $0.001 \leq n \leq 0.1$, $0.01 \leq m \leq 0.9$, with $-0.28 \leq P \leq 0.72$ and $-100 \leq s \leq 100$ as initial values, all solutions which can be found needs $P < -0.28$. The value of P increases with the increase of m and the decrease of n, but it can only go up to -0.425 when $n = 0.001$ and $m = 0.9$, which is almost the most favored parameter set within acceptable ranges for m and n. Actually m is related to the GB correction which should be small. Thus introducing one more free parameter, the GB correction, cannot change the unfavored result in case 9.

11. DGP+T_5^5+T_5^0

We contain now five parameters in total, e.g., n, $\widetilde{X}$, ν, P and s, but for simplicity, we keep only three parameters $(n, P$ and $\widetilde{X})$ free, by setting $\nu = 0.7$, $s = 1$. The reason of choosing $\nu = 0.7$ is because in case 7 the solutions give the value of ν around $0.5 \sim 1$. We find that the value of n cannot be too big, otherwise the curve will have singularity at very small z. For small n, we can find solutions without singularity, e.g., $n = 0.01$, $P = 0.020$ and

19
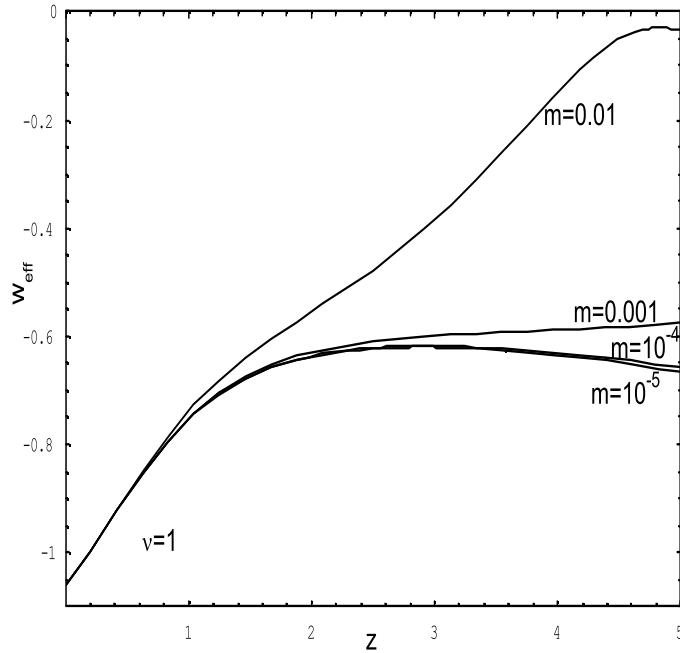
FIG. 8: $w_{eff}(z)$ curve in DGP+GB+T_5^5 case. These curves correspond to $m = 0.01$, $m = 0.001$, $m = 0.0001$ and $m = 0.00001$ respectively.

$\widetilde{X} = 78.535$. We show the proportion of different components as functions of z in Fig.9, in which $\Omega_{matter} + \Omega_{darkenergy} \equiv 1$, Ω_{matter} is the total matter as obtained from the differential equation (26); while $\Omega_{effective}$ is the effective dark energy proportion including the energy exchange effect between the bulk and brane. Since the energy exchange effect is very small (P is small), the difference between $\Omega_{effective}$ and $\Omega_{darkenergy}$ can basically be neglected. The behavior of $w_{eff}(z)$, $q(z)$ and $H(z)$ are shown in Fig.10a and Fig.10b. Different from the case 9, we see that when the bulk matter T_5^5 is considered, the modified DGP model allows the w_{eff} crossing -1 and is consistent with $H(z)$ data.

12. DGP+GB+T_5^5+T_5^0

This is the most general case in our discussion, where we have all parameters of our model: n, m, $\widetilde{X}$, ν, P and s. To simplify the calculation, we fix $\nu = 0.5$ and $s = 1$. Searching in parameters' ranges $0.01 \leq n < 10$, $0.001 \leq m < 1$, $-0.28 \leq P \leq 0.72$ and $-1000 \leq \widetilde{X} \leq 1000$, we found that when n becomes larger, the solution becomes worse, either the curves have very bad shapes or the value of P lies far away from the acceptable

20
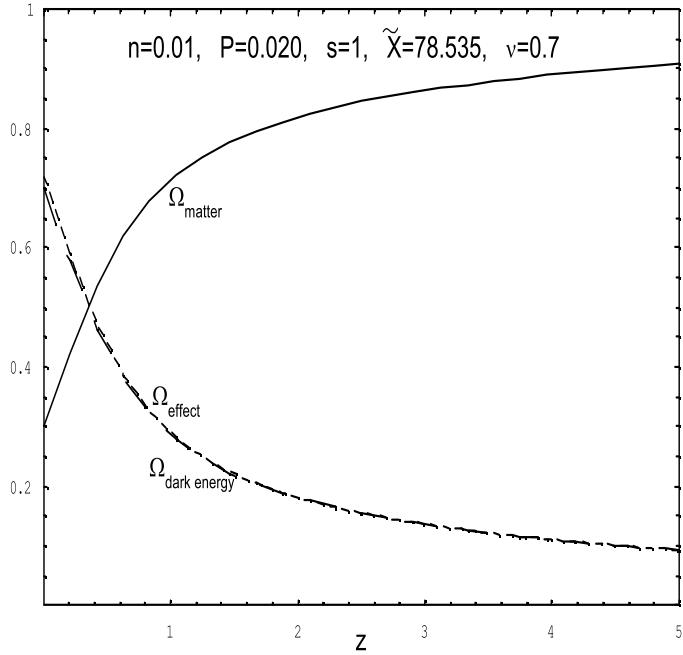
FIG. 9: Different components as functions of z in DGP+T_5^5+T_5^0 case. The solid line and the long dashed line, representing the "real" matter component and dark energy component respectively. The short dashed line shows the remainder of the total energy density after subtracting the conserved matter $\Omega_{m0}(1+z)^3 H_0^2/H(z)^2$, which acts as the effective dark energy where the energy exchange was considered. Since P is quite small, the effect of energy exchange is negligible, curves of $\Omega_{effective}$ and $\Omega_{darkenergy}$ lie almost together.

range. It is found that generally n should not be larger than 0.1. For small n, we can find the solution such as $n = 0.001$, $m = 0.01$, $P = 0.166$, $\widetilde{X} = 895.044$, whose corresponding curves of $w_{eff}(z)$, $q(z)$ and $H(z)$ are shown in Fig.11a and Fig.11b respectively.

V. CONCLUSIONS AND DISCUSSIONS

In this work we have generalized the DGP braneworld by including bulk matter content, bulk-brane energy exchange and adding the GB curvature correction term in the bulk action. We have investigated the effects of the bulk contents and the GB correction on the evolution of the universe. We have found that although the pure DGP model cannot accommodate the transition of the equation of state as indicated by recent observation, once the bulk matter
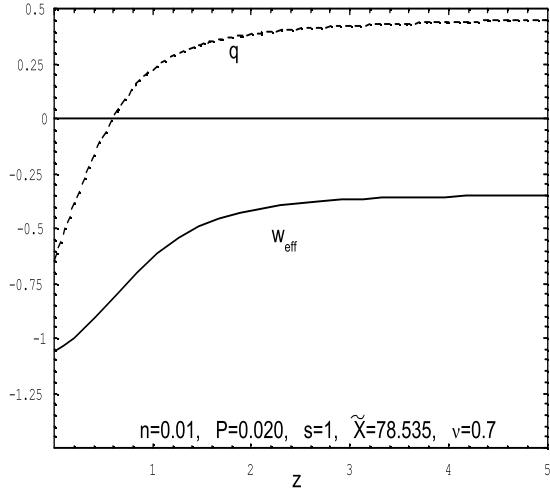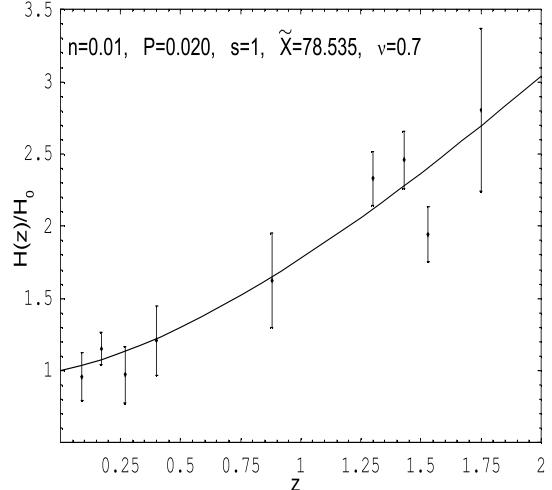
FIG. 10: $w_{eff}(z)$ and $q(z)$ curves (Fig.10a) and $H(z)$ curve (Fig.10b) in DGP+T_5^5+T_5^0 case.
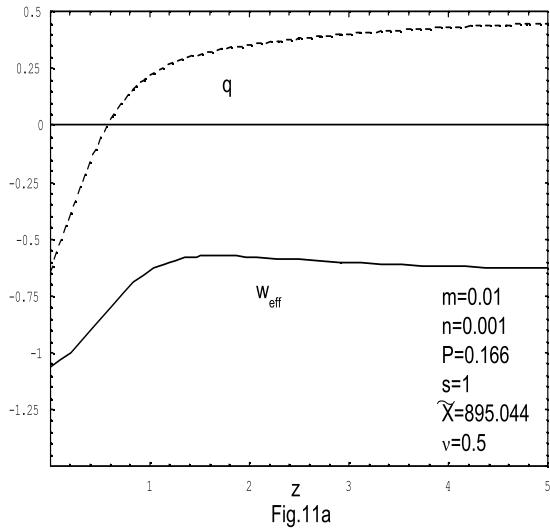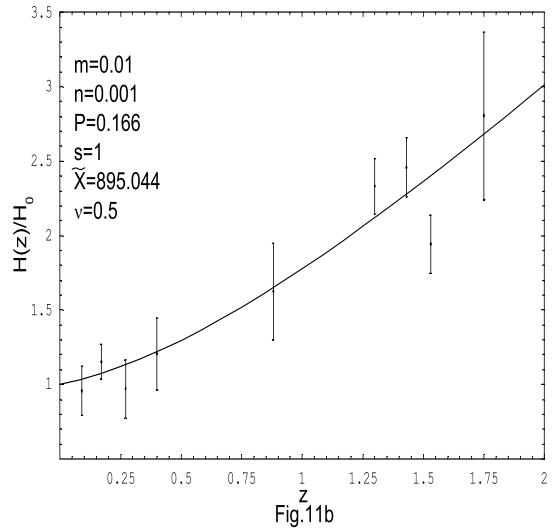
FIG. 11: $w_{eff}(z)$ and $q(z)$ curves (Fig.11a) and $H(z)$ curve (Fig.11b) in DGP+GB+T_5^5+T_5^0 case.

T_5^5 is considered, the modified model can accommodate the w_{eff} crossing -1. However this transition of the equation of state cannot be realized by just considering bulk-brane energy exchange or the GB effect but without the bulk matter contribution. Thus T_5^5 plays the major role in the modified DGP model to have the w_{eff} crossing -1 behavior. The GB term can have little influence on the late time behavior of the universe, it gives modification

22

to the equation of state at big redshift. This is because of the fact that the GB correction arises from the high energy theory, being negligible in our present cold universe. Besides the w_{eff} crossing behavior, our model can describe the Hubble parameter consistently with observation.

In our parameter space there is a generally favored range $n < 0.1$, which is crucial to have singularity free behavior in the equation of state. From the definition $n = \frac{1}{r^2 H_0^2}$, this range of n requires that the crossover factor obeys $r > 3.16 H_0^{-1}$, which is bigger than the lower bound just due to the GB correction[25, 35]. Since P and m are related, proper P requires a bit bigger value of m. The permitted value P is small. Its sign corresponds to the direction of the energy flow. The results we show previously in case 11 and case 12 have positive P standing for the influx of energy, which is considered reasonable as the explanation of the accelerating expansion of the universe for the cosmology without extra dimension. But in the brane cosmology, since we have shown with our results that the T_5^5 term dominates the effective equation of state behavior, there is no big difference whether the energy flows into or out of the brane, and in fact the solutions with negative P have also been obtained, which have similar behavior to those shown here.

From our result we see that the T_5^0 term plays little effect in the transition of equation of state, this could be due to the choice of the *ansatz*. A more general form of T_5^0 can make the calculation more difficult, since the numerical solution rather than the analytical solution of ρ from Eq. (25) will bring more difficulties in the following calculations. The solution of $\rho(a)$ should be substituted into the function of $H(z)$ after it is expressed in dimensionless notation. But in principle this is not impossible. Our method to numerically solve the nonlinear differential equation of $H(z)^2$ supplies a general way to deal with such problem, and it can relax the assumption form for T_5^0 and T_5^5. We expect to see the influence of a more general form of T_5^0 on the behavior of the equation of state of effective dark energy.

Acknowledgements

[1] A. G. Riess *et. al.*, Astronphys. J. **116**, 1009 (1998).

[2] S. Perlmutter *et. al.*, Astrophys. J. **517**, 565 (1999).

[3] A. G. Riess *et. al.*, Astrophys. J. **560**, 49 (2001).

[4] S. Weinberg, Rev. Mod. Phys. **61**, 1 (1989); N. Straumann, astro-ph/0203330; T. Padmanabhan, hep-th/0406060.

[5] R. R. Caldwell, R. Dave, and P. J. Steinhardt, Phys. Rev. Lett. **80**, 1582 (1998); P. J. E. Peebles and A. Vilenkin, Phys. Rev. D **59**, 063505 (1999); P. J. Steinhardt, L. M. Wang, and I. Zlatev, Phys. Rev. D **59**, 123504 (1999); M. Doran and J. Jaeckel, Phys. Rev. D **66**, 043519 (2002); A. R. Liddle, P. Parson, and J. D. Barrow, Phys. Rev. D **50**, 7222 (1994).

[6] A. Y. Kamenshchik, U. Moschella, and V. Pasquier, Phys. Lett. B **511**, 256 (2001).

[7] M. Li, Phys. Lett. B **603**, 1 (2004); Q. G. Huang and M. Li, JCAP **0408**, 013 (2004).

[8] R. R. Caldwell, Phys. Lett. B **545**, 23 (2002); R. R. Caldwell, M. Kamionkowski, and N. N. Weinberg, Phys. Rev. Lett. **91**, 071301 (2003); J. M. Cline, S. Y. Jeon, and G. D. Moore, Phys. Rev. D **70**, 043543 (2004).

[9] U. Alam, V. Sahni, and A. Starobinsky, JCAP **0406**, 008 (2004); Y. G. Gong, Class. Quant. Grav. **22**, 2121 (2005); Y. Wang and M. Tegmark, Phys. Rev. D **71**, 103513 (2005); Y. Wang and P. Mukherjee, Astrophys. J. **606**, 654 (2004); R. Daly and S. Djorgovski, Astrophys. J. **612**, 652 (2004); U. Alam, V. Sahni, T. Saini, and A. Starobinsky, Mon. Not. Roy. Astron. Soc. **354**, 275 (2004); T. Choudhury and T. Padmanabhan, Astron. Astrophys. **429**, 807 (2005).

[10] B. Feng, X. L.Wang, and X. M. Zhang, Phys. Lett. B **607**, 35 (2005); W. Hu, Phys. Rev. D **71**, 047301 (2005); Z. K. Guo, Y. S. Piao, X. M. Zhang, and Y. Z. Zhang, Phys. Lett. B **608**, 177 (2005); X. F. Zhang, H. Li, Y. S. Piao, and X. Zhang, astro-ph/0501652.

[11] M. Z. Li, B. Feng, and X. M. Zhang, hep-ph/0503268.

[12] B. Wang, Y. Gong, and E. Abdalla, Phys. Lett. B **624**, 141 (2006); B. Wang, C. Y. Lin, and E. Abdalla, Phys. Lett. B **637**, 357 (2006); B. Wang, J. D. Zang, C. Y. Lin, E. Abdalla and S. Micheletti, Nucl.Phys.B778, 69,2007, astro-ph/0607126.

[13] S. Nojiri and S. D. Odintsov, Phys. Lett. B **562**, 147 (2003); S. Nojiri and S. D. Odintsov, Phys. Rev. D **70**, 103522 (2004); A. Vikman, Phys. Rev. D **71**, 023515 (2005); A. Anisi-

mov, E. Babichev, and A. Vikman, JCAP **0506**, 006 (2005); S. Nojiri, S. D. Odintsov, and S. Tsujikawa, Phys. Rev. D **71**, 063004 (2005); P. Singh, gr-qc/0502086; H. Stefancic, astro-ph/0504518; E. O. Kahya and V. K. Onemli, gr-qc/0612026; T. Koivisto and D. F. Mota, Phys. Lett. B **644**, 104 (2007), astro-ph/0606078; T. Koivisto and D. F. Mota, Phys. Rev. D **75**, 023518 (2007), hep-th/0609155; V.K. Onemli, R.P. Woodard, Phys.Rev.D 70, 107301 (2004).

[14] G. R. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999); Phys. Rev. Lett. **83**, 4690 (1999).

[15] E. Kiritsis, G. Kofinas, N. Tetradis, T. N. Tomaras, and V. Zarikas, JHEP **0302**, (2003) 035; E. Kiritsis, N. Tetradis, and T. N. Tomaras, JHEP **0203**, (2002) 019; P. S. Apostolopoulos and N. Tetradis, Phys. Rev. D **71**, 043506 (2005); P. S. Apostolopoulos and N. Tetradis, Phys. Lett. B **633**, 409 (2006); E. Kiritsis, JCAP **0510**, 014 (2005); K. I. Umezu, K. Ichiki, T. Kajino, G. J. Mathews, R. Nakamura, and M. Yahiro, Phys. Rev. D **73**, 063527 (2006); M.R. Setare, Phys.Lett.B642,421,2006.

[16] R. G. Cai, Y. Gong, and B. Wang, JCAP **0603**, 006 (2006).

[17] P. S. Apostolopoulos and N. Tetradis, hep-th/0604014; P. S. Apostolopoulos, N. Brouzakis, N. Tetradis, E. Tzavara, arXiv:0708.0469.

[18] C. Bogdanos and K. Tamvakis, hep-th/0609100; C. Bogdanos, A. Dimitridis, and K. Tamvakis, hep-th/0611094.

[19] A. Sheykhi, B. Wang, and N. Riazi, Phys. Rev. D **75**, 123513 (2007).

[20] B. Zwiebach, Phys. Lett. B **156**, 315 (1985); D. G. Boulware and S. Deser, Phys. Rev. Lett. **55**, 2656 (1985).

[21] D. Lovelock, J. Math. Phys. **12**, 498 (1971).

[22] G. Kofinas, R. Maartens, and E. Papantonopoulos, JHEP **0310**, 066 (2003).

[23] G. Dvali, G. Gabadadze, and M. Porrati, Phys. Lett. B. **485**, 208 (2000).

[24] G. Kofinas, G. Panotopoulos, and N. Tomaras, hep-th/0510207.

[25] R. A. Brown, R. Maartens, E. Papantonopoulos, and V. Zamarias, gr-qc/0508116.

[26] R. G. Cai, H. S. Zhang, and A. Wang, hep-th/0505186.

[27] C. Deffayet, Phys. Lett. B. **502**, 199 (2001).

[28] C. Barceló, C. Germani, and C. F. Sopuerta, Phys. Rev. D **68**, 104007 (2003).

[29] R. Maartens, Phys. Rev. D **62**, 084023 (2000).

[30] D. N. Spergel *et al.* [WMAP Collaboration], Astrophys. J. Suppl. **148**, 175 (2003).

[31] E. V. Linder and A. Jenkins, Mon. Not. Roy. Astron. Soc. **346**, 573 (2003).

[32] R. Jimenez, L. Verde, T. Treu, and D. Stern, Astrophys. J. **593**, 622 (2003).

[33] J. Simon, L. Verde, and R. Jimenez, Phys. Rev. D **71**, 123001 (2005).

[34] W. L. Freeman, *et al.*, astro-ph/0012376.

[35] J. H. He, B. Wang, and E. Papantonopoulos, arXiv:0707.1180, Phys. Lett. B (in press).

n = 0.046,   X = 2.729,   ν = 0.948

Fig.3b